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METHOD AND BUSINESS PROCESS FOR THE ESTIMATION OF EROSION COSTS IN ASSEMBLE-TO-ORDER MANUFACTURING OPERATIONS

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Related Applications

This application is related to the following commonly owned concurrently filed application: "Method and Business Process for the Estimation of Mean Production for Assemble-To-Order Manufacturing Operations." That document is incorporated in this application by reference.

This application is a continuation of US Provisional Application Serial Number 60/229,611 filed June 21, 2000. That disclosure is incorporated in this document by reference.

Technical Field of the Invention

The present invention is generally related to resource-planning methods used by manufacturing companies and other organizations. More specifically, it relates to risk management in the procurement of unfinished goods by estimation of expected excess component inventory and erosion costs.

Background of the Invention

In the field of product manufacturing, the term assemble-to-order (ATO) refers to systems where products are not produced until the demand for the products becomes known. Manufacture-to-stock (MTS) systems, on the other hand, base production on component availability. Unlike ATO manufacturing, MTS systems often produce products before the demand for those products has been determined.

Manufacturing companies typically prefer to use ATO manufacturing operations whenever possible. ATO allows manufacturers to minimize their component inventories. This reduces the amount of capital invested in components and reduces the risk that components will loose value or become obsolete before they are transformed and sold as products.

In an ATO operation, the availability of components is the key variable that controls the ability to produce. If component availability is limited (either because inventory is limited or because supplier commitments to ship the component on demand are insufficient) then fulfillment of demand either fails or is delayed. On the other hand, if excess inventory is left over, it must be liquidated or utilized later. Both outcomes are costly to the company. If the company liquidates, it may be forced to do so at substantially reduced prices. Many items lose value because of innovation. This is especially true in the especially in the fast moving high-tech sector where goods, such as microprocessor are continually superseded with cheaper more powerful replacements. In other industries there are shelf life considerations that make it costly to try to utilize excess inventory at a later time. In both cases there is capital tied up in extra inventory and that corresponds to incurring a cost equivalent to the firms funding rate.

The cost incurred for component inventory that exceeds the inventory required to meet demand is referred to as the erosion cost. The estimation of erosion costs is paramount in making intelligent business decisions about the proper amount of components to obtain commitments in any given period. This is particularly true when a large number of products are made from a large number of the same components.

Summary of the Invention

An embodiment of the present invention includes a method for the computation of surplus components. For this method, a planner first identifies each component required to produce a product. For each component, the planner defines a planned level and an uncancelable level. The planned level for a component is the quantity at which the component is expected to be available. The uncancelable level for a component is the quantity of the component that cannot be liquidated without charge. The planner also defines a vector of connect rates for the components.

After the required data has been entered, an expected surplus is computed for each component. To compute a component's expected surplus, the component is assumed to be available at its uncancelable level. The remainder of the components are assumed to be available at their respective planned levels. The mean production for the component is computed and used, along with the uncancelable level the selected component and the vector of connect rates to compute the component's expected surplus. This computation is repeated for all components in turn.

The expected component surpluses can then be used to compute the expected erosion cost per component and the total erosion cost for all components.

Other aspects and advantages of the present invention will become apparent from the following descriptions and accompanying drawings.

Brief Description of the Drawings

For a more complete understanding of the present invention and for further features and advantages, reference is now made to the following description taken in conjunction with the accompanying drawings, in which:

Figure 1 is a block diagram of a computer system shown as a representative environment for deployment of the present invention.

Figure 2 is a flowchart showing the steps associated with an embodiment of the method for computing expected surplus of the present invention.

Figure 3 is a flowchart showing the steps associated with an embodiment of the method for computing expected erosion costs per component of the present invention.

Figure 4 is a flowchart showing the steps associated with an embodiment of the method for computing total expected erosion cost of the present invention.

Detailed Description of the Preferred Embodiments

The preferred embodiments of the present invention and their advantages are best understood by referring to Figures 1 through 4 of the drawings. Like numerals are used for like and corresponding parts of the various drawings.

Definitions

Component plan: a list of quantities for each component, representing a company's component order for a given planning period.

Scenario: a set of assumptions about products and components. A scenario includes product parameters, component parameters, component consumption, component interactions, and an allocation policy.

Environment

In Figure 1, a computer system 100 is shown as a representative environment for an embodiment of the present invention. Structurally, computer system 100 includes a processor, or processors 102, and a memory 104. Processor 102 can be selected from a wide range of commercially available or custom types. An input device 106 and an output device 108 are connected to processor 102 and memory 104. Input device 106 and output device 108 represent all types of I/O devices such as disk drives, keyboards, modems, network adapters, printers and displays. Computer system 100 may also includes a disk drive 110 of any suitable disk drive type (equivalently, disk drive 110 may be any non-volatile mass storage system such as "flash" memory).

Computing Expected Surplus

As shown in Figure 2, an embodiment of the present invention includes a Method 200 for the computation of surplus components (expected excess number of components remaining at the end of a period). Method 200 consists of the following steps:

In step 202 a user enters data describing one or more products and their associated components. The data entered includes all the data required for mean production computation as disclosed in the related application "Method and Business Process for the Estimation of Mean

Production for Assemble-To-Order Manufacturing Operations." In particular, this includes, for each product, information describing:

1) The type of components required to produce the product,

2) The number of each component required (A),

3) Component allocations (d),

4) The mean demand for the product (μ),

5) The standard deviation of the demand for the product,

6) The correlations between the product and any other product.

This information can be input manually by the user or retrieved from a user database or other source.

For each component i, the value d_i (referred to above as component allocations) represents the maximum number of that is available (i.e. the expeditable amounts specified for the mean production computations). In many cases, positioned components may be returning or canceled. In these cases, only the uncancelable portion is subject to erosion. As part of step 202, the user enters (or otherwise inputs) a value d_i^0 to represent the uncancelable portion of each component of interest. For each component i, the quantity $d_i - d_i^0$ represents the number that can be liquidated or returned at no cost. For some components, return or liquidation of any amount has an associated cost. For each component i, of this type the value d_i^0 is equal to d_i.

Steps 204 through 214 form a loop. This loop sequences each component i for which the calculation of surplus components is desired.

In step 206, each component plan d_i is set to its original value. This means that each component is assumed to be available in its planned quantity.

In step 208 the component plan for component i (i.e., d_i) is set to d_i^0. This means that component i is assumed to be available at the uncancelable level for that component.

In step 210 the component levels specified in steps 206 and 208 (i.e., d_i^0 for component i and d_i for all other components) are used to compute an expected mean production q_i^0 for a specified planning period. In general, there are several methods, such as Monte Carlo simulation that may be used to perform this computation. Preferably, but not necessarily, the method disclosed in the related application "Method and Business Process for the Estimation of Mean Production for Assemble-To-Order Manufacturing Operations."

In step 212 the formula $e_i = d_i^0 - a_i \cdot q_i^0$ is used to compute the expected number of erodible components i. In this formula d_i^0 is the uncancelable level for component i, a_i is the vector of connect rates for component i, and q_i^0 is the expected mean production computed in step 210.

In step 216 the expected excess is reported to the user or returned to the system for further use.

Computing Expected Erosion Costs Per Component

As shown in Figure 4, an embodiment of the present invention includes a Method 400 for the computing the expected erosion costs per component (the expected financial cost of having leftover inventory for a subset of components). Method 400 consists of the following steps:

In step 402 a user inputs the erosion costs per surplus component.

In step 404 the expected surplus is evaluated using Method 200. This step may be done in parallel with step 402.

In step 406 the expected surplus is multiplied by the per-unit erosion costs to obtain the expected erosion cost for each component of interest.

In step 408 the expected erosion cost for each component of interest is reported to the user.

Computing Total Expected Erosion Cost

As shown in Figure 5, an embodiment of the present invention includes a Method 500 for computing the total expected erosion cost (i.e., estimating the total erosion cost a company should expect given a certain plan for component availability). Method 500 consists of the following steps:

In step 502 the component-by-component expected erosion costs are evaluated for all components of the planning portfolio. This step may be done in parallel.

In step 504 the numbers obtained in step 502 are summed.

In step 506 the single expected erosion cost number is reported back to the user, or stored for further processing by the system.

The method of computing erosion costs can be used as the basis for computing expected cancellation fees using the following equation:

$$\left\langle C_i^{(l)} \right\rangle = \left\langle E_i^{(l)} \right\rangle - \left\langle E_i^{(l-1)} \right\rangle$$

The method of computing erosion costs can be used as the basis for computing expected expediting fees using the following equation:

$$\left\langle X_i^{(l)} \right\rangle = \left(d_i^{(l)} - d_i^{(l-1)} \right) - \left(\left\langle E_i^{(l)} \right\rangle - \left\langle E_i^{(l-1)} \right\rangle \right)$$

Although particular embodiments of the present invention have been shown and described, it will be obvious to those skilled in the art that changes and modifications may be made without departing from the present invention in its broader aspects, and therefore, the appended claims are to encompass within their scope all such changes and modifications that fall within the true scope of the present invention.

1 Introduction

When component stock exceeds component consumption a surplus remains. When the producer does not have the option to return unused components, the surplus is subject to *value erosion*, in at least three ways:

- opportunity cost of capital tied up in hard assets;
- technical erosion due to design changes and technological progress; and
- physical erosion due to spoilage.

As a result, the value of a surplus component is strictly less than its original value when it was purchased.

In some cases, the supply contract includes an option to return all or some of the component surplus subject to a *cancellation fee*. In the following analysis, we will assume that the cancellation fee is strictly less than the associated value erosion so that it will always be rational to exercise the cancellation option when it is available.

Similarly, a supply contract may also include an option to *expedite* extra components when component demand exceeds the stock. Again, expediting will carry with it an *expediting fee* which can be thought of as a premium for rush service. Just as there is a lower limit to how many components the supplier is willing to take back, there is also an upper limit beyond which components cannot be expedited at any cost. When demand exceeds component availability the company may charge itself a "penalty" for missed (or delayed) sales.

In [1], Dagum introduced a model of erosion and expediting based on a simplifying assumption that a distribution on component demand could be induced from product demand without regard to production policies. In this report we relax that assumption, recognizing that component consumption is not driven by product demand directly, but rather by production.

2 Simple Erosion

In this section we consider the simple case of erosion when the supply contract does not allow for cancellation or expediting. As usual, we define product demand $\mathbf{x} \in \Re^n$ as a random variable distributed normally with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$. Given production $\mathbf{q} \in \Re^n$ and connect rate matrix $\mathbf{A} \in \Re^{m \times n}$, induced component consumption is $\mathbf{Aq}$. We denote the available component stock by $\mathbf{d} \in \Re^m$. Thus, the volume of component i subject to erosion is

$$e_i = \max(d_i - [\mathbf{Aq}]_i, 0) \equiv (d_i - [\mathbf{Aq}]_i)^+. \tag{2.1}$$

Component supply $\mathbf{d}$ defines a *feasible region* Ω, in which component supply is sufficient to meet demand. The feasible region Ω is a defined as a polyhedral set

$$\Omega = \cap_{i \in \mathcal{M}} H_i, \tag{2.2}$$

where $\mathcal{M}$ denotes the set of relevant components, and H_i denotes the halfspace where component supply $\mathbf{d}$ is sufficient to meet demand, i.e.,

$$H_i = \{\mathbf{x} : d_i - [\mathbf{A}\mathbf{x}]_i \geq 0\}. \tag{2.3}$$

In general, the feasible region can be product specific. In other words, the feasible region from the perspective of product j may be different that that of product k. We denote the feasible region with respect to product j as $\Omega(j)$, and the set of relevant components as $\mathcal{M}(j)$. In particular, under uniform allocation $\mathcal{M}(j)$ is the *Markov Blanket* associated with product j, where

$$\mathcal{M}(j) = \bigcup_{i=\mathcal{D}} \tag{2.4}$$

Under prioritized allocation feasibility The production function $\mathbf{q} : \mathbf{x} \in \Re^n \rightarrow \mathbf{q} \in \Omega$ establishes a level of feasible production given product demand $\mathbf{x}$ and component availability $\mathbf{d}$. In principle, it is desirable to produce as much as is feasible, but never more than what is demanded.

Therefore, when demand is feasible production maps demand to itself. When demand is not feasible, production maps infeasible demand to the *feasible frontier*

$$\partial\Omega = \Omega \cap (\cup_i \partial H_i) \tag{2.5}$$

where $\partial H_i = \{\mathbf{x} : d_i - [\mathbf{A}\mathbf{x}]_i = 0\}$. [1] Thus production is defined as follows

$$\mathbf{q}(\mathbf{x}, \mathbf{d}) = \begin{cases} \mathbf{x}, & \text{if } \mathbf{x} \in \Omega, \\ \mathbf{q} \in \partial\Omega & \text{otherwise.} \end{cases} \tag{2.6}$$

Expected Erosion. Expected erosion is defined as

$$\langle E_i \rangle = \int (\mathbf{d}_i - [\mathbf{A}\mathbf{q}]_i)^+ f(\mathbf{q})\, d\mathbf{q} \tag{2.7}$$

Unfortunately, we do not have an explicit representation of the probability density function

[1] For clarity we will also denote the i-th row of $\mathbf{A}$ by $\mathbf{a}_i$

$f(\mathbf{q})$. However, it is useful to recognize that by the definition of production in (2.6), $\mathbf{q} \in \Omega$, and therefore $d_i - [\mathbf{A}\mathbf{q}(\mathbf{x}, \mathbf{d})]_i \geq 0$ for all $i \in [m]$. Thus, expected erosion for component i can be computed as

$$\begin{aligned}\langle E_i \rangle &= \int (\mathbf{d}_i - [\mathbf{A}\mathbf{q}]_i)^+ f(\mathbf{q})\, d\mathbf{q} \\ &= \int (\mathbf{d}_i - \mathbf{a}_i \cdot \mathbf{q}(\mathbf{x}, \mathbf{d}))\, f(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})\, d\mathbf{x} \\ &= \mathbf{d}_i - \mathbf{a}_i \cdot \langle \mathbf{q} \rangle . \end{aligned} \tag{2.8}$$

In this simple case, computation of expected erosion, reduces to computation of expected production $\langle \mathbf{q} \rangle$. Kralik and Dagum [2] present a method to compute expected production under the distributed production function

$$\mathbf{q}(\mathbf{x}, \mathbf{d}) = \mathbf{x} - \alpha \mathbf{u}, \quad \text{where} \tag{2.9}$$

$$\alpha(\mathbf{x}, \mathbf{d}) = \max_i \left(\frac{d_i - \mathbf{a}_i \cdot \mathbf{x}}{\mathbf{a}_i \cdot \mathbf{u}}, 0 \right) . \tag{2.10}$$

When $\mathbf{u} = \mathbf{x}$, this function satisfies the gate mapping property. Other values for u, i.e. $\mathbf{u} = \boldsymbol{\mu}$, may not satisfy the gate mapping property over all $\mathbf{x} \in \Re^n$, but may represent reasonable approximations.

3 Erosion net of cancellation

In some cases, the producer has the option to return or *cancel* positioned components. In this case, only the *uncancellable* portion of component surplus will be subject to erosion. We denote the uncancellable portion of available components by $\mathbf{d}^{(0)}$. We continue to denote the full component availability as $\mathbf{d}$ and production $\mathbf{q} \equiv \mathbf{q}(\mathbf{x}, \mathbf{d})$, but we introduce the following additional notation

$$\begin{aligned}\mathbf{d}_i^{(0)} &\equiv \left[d_i \quad d_2 \quad \ldots \quad d_i^{(0)} \quad \ldots \quad d_m \right] \\ H_i^{(0)} &\equiv \left\{ \mathbf{x} : d_i^{(0)} - [\mathbf{A}\mathbf{x}]_i \geq 0 \right\} \\ \Omega_i^{(0)} &\equiv \Omega \cap H_i^{(0)} \\ \mathbf{q}_i^{(0)} &\equiv \mathbf{q}\left(\mathbf{x}, \mathbf{d}_i^{(0)}\right) \end{aligned}$$

In view of the above notation, a more precise definition of erosion is given by

$$E_i^{(0)} = \left(d_i^{(0)} - [\mathbf{A}\mathbf{q}]_i \right)^+ \tag{3.1}$$

Taking the expectation of $E_i^{(0)}$ in (3.1) presents a complication because production $\mathbf{q}$ is defined by the maximum component availability levels $\mathbf{d}$, while erosion is computed with respect to the uncancellable levels $d_i^{(0)}$. Nevertheless, we will show that a simplification similar to (2.8) exists for a class of production functions which we refer to as *gate mapping*.

Gate Mapping Production. Given demand x, we define the *gating* component $i \in [m]$, to be the component for which the plan d_i represents a binding constraint on production, *i.e.* production $\mathbf{q}$ falls on the d_i-face of the feasible region. In order to proceed, consider the function

$$g_i(\mathbf{x}, \mathbf{d}) = \mathbf{a}_i \cdot \mathbf{q}(\mathbf{x}, \mathbf{d}) \tag{3.2}$$

$$= \begin{cases} \mathbf{a}_i \cdot \mathbf{x} & \text{if} \quad \mathbf{x} \in \Omega \\ d_i & \text{if} \quad \mathbf{x} \in \bar{\Omega} \quad \text{and } i \text{ is gating} \\ \tilde{g}_i(\mathbf{x}, \mathbf{d}) \leq d_i & \text{otherwise.} \end{cases} \tag{3.3}$$

In the above definition, $\tilde{g}_i$ is some function which depends on the problem details. Its region of definition Ψ_i is the intersection of $\bar{\Omega}$ with the complement of the region where i is gating.

For the usual local-u production policy [2], $\tilde{g}_i$ has an additional property: in its region of definition, it is independent of d_i. This is the *gate mapping* property:

Definition 3.1 *Gate mapping property: an allocation policy is* gate mapping *if the following condition holds for every* $\mathbf{x}$*: If for both* $\mathbf{d} = (d_1, \ldots, d_i, \ldots, d_n)$ *and* $\mathbf{d}' = (d_1, \ldots, d_i', \ldots, d_n)$ *the gating component is the same* $j \neq i$*, then* $\mathbf{q}(\mathbf{x}, \mathbf{d}) = \mathbf{q}(\mathbf{x}, \mathbf{d}')$.

Lemma 3.2 *When production* $\mathbf{q}$ *is* gate mapping *as defined above, the production mappings* $\mathbf{q}$ *and* $\mathbf{q}_i^{(0)}$ *are equivalent with respect to demand for which there is erosion associated with component* i*. In math:* $\mathbf{q} = \mathbf{q}_i^{(0)}$ *for all* $\mathbf{x} \in \left\{\mathbf{x} : d_i^{(0)} - [\mathbf{A}\mathbf{q}]_i > 0\right\}$.

Proof. There are two cases in which the condition $d_i^{(0)} > [\mathbf{A}\mathbf{q}]_i$ holds:

1. $\mathbf{x} \in \Omega_i^{(0)} \subset \Omega$, or
2. $\mathbf{q}$ maps $\mathbf{x} \in \bar{\Omega}$ to the feasible frontier $\partial\Omega$ such that $\mathbf{q} \in H_i^{(0)}$.

In the first case, $\mathbf{q} = \mathbf{q}_i^{(0)} = \mathbf{x}$ because under the extra constraint $\mathbf{x} \in H_i^{(0)}$ had no effect on feasibility. In the second case, there exists a component $k \neq i$ for which the feasibility constraint $\mathbf{x} \in Hk$ is more constraining than the new constraint $\mathbf{x} \in H_i^{(0)}$. Since production is *gate mapping*, infeasible demand is mapped to the part of the feasible frontier ∂Hk associated with the gating component k. Since the new constraint is not binding, it does not not change production mapping. ■

Lemma 3.3 *If there is no erosion associated with demand* $\mathbf{x}$ *when it is mapped by a gate mapping function* $\mathbf{q}$*, there is also no erosion in component* i *when production is mapped by* $\mathbf{q}_i^{(0)}$.

Proof. Let $\Phi = \left\{\mathbf{x} : d_i^{(0)} - \mathbf{a}_i \cdot \mathbf{q}(\mathbf{x}, \mathbf{d})]_i \leq 0\right\}$. Because production is gate mapping, $\mathbf{q}$ maps $\mathbf{x} \in \Phi$ to a point $\mathbf{q} \notin H_i^{(0)}$. Therefore, when the feasible region is constrained to $\Omega_i^{(0)}$ by adding the constraint $\mathbf{q} \in H_i^{(0)}$, component i becomes gating for all $\mathbf{x} \in \Phi$ and therefore $\mathbf{q}_i^{(0)}$ will map that demand to the face of $\partial\Omega_i^{(0)}$ defined by the hyperplane $\partial H_i^{(0)} = \left\{\mathbf{q} : d_i^{(0)} - [\mathbf{A}\mathbf{q}]_i = 0\right\}$ along which there is no erosion. ■

Lemma 3.4 *For all* $\mathbf{x} \in \Re^n$, $d_i^{(0)} - [\mathbf{A}\mathbf{q}]_i = d_i^{(0)} - [\mathbf{A}\mathbf{q}_i^{(0)}]_i$.

Proof. The proof follows directly from Lemmas 3.2 and 3.3. From Lemma 3.2 we see that when there is erosion, the production mapping $\mathbf{q} = \mathbf{q}_i^{(0)}$. From Lemma 3.3 we see that when there is no erosion under $\mathbf{q}$, the new mapping $\mathbf{q}_i^{(0)}$ maps demand to the hyperplane $\partial H_i^{(0)}$ where there is no erosion. ■

We now state the following two propositions:

Proposition 3.1 *When production is gate mapping property, expected erosion for component* i *can be computed as*

$$\left\langle E_i^{(0)} \right\rangle = d_i^{(0)} - \mathbf{a}_i \cdot \left\langle \mathbf{q}_i^{(0)} \right\rangle. \tag{3.4}$$

Proof. This proposition follows directly from Lemma 3.4 and Equation (2.8). ■

Proposition 3.2 *The production function defined in* (2.9) *and* (2.10) *is gate mapping.*

Proof. The max function in (2.10) ensures that demand is mapped back to hyperplane associated with the gating component. ■

Therefore we can compute (3.4) using available methods.

4 Cancellation

In the previous section we demonstrated how to compute the expected volume of erodible components. In this section we consider the expected volume of cancelled components. Cancelled components are the surplus components that can be returned to the supplier and therefore are not subject to erosion.

4.1 Simple cancellation

As before, we denote the uncancellable volume of components by $\mathbf{d}^{(0)}$. We now denote the component plan by $\mathbf{d}^{(1)}$, where the component plan represents the amount of component positioned. Cancellation only applies to components that have been positioned. We continue to denote available components (which may include expediting) by $\mathbf{d}$.

Given a production $\mathbf{q}$ such that $\mathbf{q} \in H_i^{(1)}$, the number of components i that can be cancelled is

$$C_i = d_i^{(1)} - \max(d_i^{(0)}, [\mathbf{Aq}]_i). \tag{4.1}$$

The cancellation is otherwise 0.

Proposition 4.1 *When production is gate mapping, expected cancellation can be computed from two erosion computations:*

$$\langle C_i \rangle = \left\langle E_i^{(1)} \right\rangle - \left\langle E_i^{(0)} \right\rangle, \tag{4.2}$$

where

$$\left\langle E_i^{(l)} \right\rangle \equiv d_i^{(l)} - \mathbf{a}_i \cdot \left\langle \mathbf{q}_i^{(l)} \right\rangle.$$

Proof. Expanding the definition in (4.1), yields

$$C_i = \begin{cases} d_i^{(1)} - [\mathbf{Aq}]_i - (d_i^{(0)} - [\mathbf{Aq}]_i), & \text{if } \mathbf{q} \in H_i^{(0)}, \\ d_i^{(1)} - [\mathbf{Aq}]_i & \text{if } d_i^{(0)} < [\mathbf{Aq}]_i < d_i^{(1)}, \\ 0 & \text{otherwise.} \end{cases}$$

Taking the expected value of $\langle C_i \rangle$ over $\mathbf{x} \in \Re^n$ implies that

$$\langle C_i \rangle = \left\langle \left(d_i^{(1)} - [\mathbf{Aq}]_i \right)^+ \right\rangle - \left\langle \left(d_i^{(0)} - [\mathbf{Aq}]_i \right)^+ \right\rangle \tag{4.3}$$

We recognize the first term in (4.3) as simple erosion equivalent to $\left\langle E_i^{(1)} \right\rangle$. We recognize that the second term in (4.3) is equivalent to $\left\langle E_i^{(0)} \right\rangle$ following the discussion of erosion with cancellation in § 3. ■

4.2 Multilevel cancellation

In principle, there may be many levels of cancellation (or expediting). We denote a cancellation or expediting level by l, where $0 \leq l \leq L$, and the associated volumes of components by $\mathbf{d}^{(l)}$, where $d_i^{(0)} < \ldots < d_i^{(l)} < \ldots d_i^{(L)}$. Thus $\mathbf{d} = \mathbf{d}^{(L)}$ represents components available for production. We can now expand the analysis to multiple levels of cancellation and introduce the following expanded notation:

$$\begin{aligned}
\mathbf{d}^{(l)} &\equiv \left[d_i^{(l)} \quad \ldots \quad d_m^{(l)}\right], \\
H^{(l)} &\equiv \left\{\mathbf{x} : d_i^{(l)} - [\mathbf{A}\mathbf{x}]_i \geq 0\right\}, \\
\Omega^{(l)} &\equiv \cap_{i \in [m]} H_i^{(l)}, \text{ and} \\
\mathbf{q}^{(l)} &\equiv \mathbf{q}\left(\mathbf{x}, \mathbf{d}^{(l)}\right).
\end{aligned}$$

As before

$$\begin{aligned}
\mathbf{d} &\equiv \mathbf{d}^{(L)}, \\
H_i &\equiv H^{(L)}, \\
\Omega &\equiv \Omega^{(L)}, \text{ and} \\
\mathbf{q} &\equiv \mathbf{q}^{(L)}.
\end{aligned}$$

Following the reasoning in § 4.1, expected cancellation of level $0 < l < L$ components is simply

$$\left\langle C_i^{(l)} \right\rangle = \left\langle E_i^{(l)} \right\rangle - \left\langle E_i^{(l-1)} \right\rangle. \tag{4.4}$$

5 Expediting

Expediting is essentially the mirror problem to cancellation. However, instead of computing component surplus, we are computing component use. Thus while cancellation measured the nonnegative distance to an upper boundary, expediting measures then nonnegative distance to a lower boundary.

Let $\mathbf{d}^{(l)}$ represent the components available at expediting level l. We now define expediting as

associated with component i as

$$X_i^{(l)} = \begin{cases} \min\left([\mathbf{Aq}]_i, d_i^{(l)}\right) - d_i^{(l-1)}, & \text{if } \mathbf{q} \in \bar{H}_i^{(l-1)} \\ 0, & \text{otherwise.} \end{cases} \tag{5.1}$$

Proposition 5.1 *When production is gate mapping, expected expediting can be computed as*

$$\left\langle X_i^{(l)} \right\rangle = \left(d_i^{(l)} - d_i^{(l-1)}\right) - \left(\left\langle E_i^{(l)} \right\rangle - \left\langle E_i^{(l-1)} \right\rangle\right). \tag{5.2}$$

Proof. We follow the same logic as before. Expanding the definition in (5.1), yields

$$X_i^{(l)} = \begin{cases} ([\mathbf{Aq}]_i - d_i^{(l-1)}) - [\mathbf{Aq}]_i - (d_i^{(l)}), & \text{if } \mathbf{q} \in \bar{H}_i^{(l)}, \\ [\mathbf{Aq}]_i - d_i^{(l-1)}, & \text{if } d_i^{(l-1)} < [\mathbf{Aq}]_i < d_i^{(l)}, \\ 0, & \text{otherwise.} \end{cases}$$

Taking the expected value of $X_i^{(l)}$ yields

$$\left\langle X_i^{(l)} \right\rangle = \left\langle \left([\mathbf{Aq}]_i - d_i^{(l-1)}\right)^+ \right\rangle - \left\langle \left([\mathbf{Aq}]_i - d_i^{(l)}\right)^+ \right\rangle.$$

With a simple transformation we can recognize these as already known quantities:

$$\begin{aligned} \left\langle \left([\mathbf{Aq}]_i - d_i^{(l)}\right)^+ \right\rangle &= \left\langle [\mathbf{Aq}]_i - d_i^{(l)} \right\rangle + \left\langle \left(d_i^{(l)} - [\mathbf{Aq}]_i\right)^+ \right\rangle \\ &= \mathbf{a}_i \cdot \langle \mathbf{q} \rangle - d_i^{(l)} + \left\langle E_i^{(l)} \right\rangle \end{aligned}$$

With this, we can write

$$\left\langle X_i^{(l)} \right\rangle = \left(d_i^{(l)} - d_i^{(l-1)}\right) - \left(\left\langle E_i^{(l)} \right\rangle - \left\langle E_i^{(l-1)} \right\rangle\right). \tag{5.3}$$

■

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[1] Paul Dagum. Asset risk management. Technical Report TR-03-99, Rapt Technologies Corporation, 324 Ritch Street, San Francisco, CA 94107, February 1999.

[2] Paul Dagum, Michael Goldbach, and Balazs Kralik. Demand statistical arbitrage with variants of the uniform production policy. Technical Report TR-14-00, Rapt Technologies Corporation, 324 Ritch Street, San Francisco, CA 94107, January 2000.

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Appendix A:

Invention Disclosure

ID-06-00

Title: **Method and business process for the estimation of erosion costs in assemble-to-order manufacturing operations**

Inventor(s): **Balazs Kralik, Michael Goldbach, and Paul Dagum**

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Background of the invention

Problem definition

When possible, it is beneficial for manufacturing companies to produce their outputs in an assemble-to-order (ATO) fashion. While manufacture-to-stock (MTS) manufacturers produce outputs before demand for them is revealed, ATO operations only produce outputs as the order stream arrives.

In an ATO operation, the availability of components is the key variable that controls the ability to produce. If component availability is limited (either because inventory is limited or because supplier commitments to ship the component on demand are insufficient), then fulfillment of demand either fails or is delayed.

On the other hand if excess inventory is left over, it must be liquidated or utilized later. Both outcomes are costly to the company. If the company liquidates, it can only do so at substantially reduced prices. Many items especially in the fast-moving high-tech sector lose value because of innovation (e.g. who wants last year's microprocessor when this year's is twice as fast for the same price). In other industries there are simple shelf life considerations that make it costly to try to utilize excess inventory at a later time. In addition, in both cases there is capital tied up in the extra inventory and that corresponds to incurring a cost equivalent to the firms funding rate.

The estimation of the erosion costs is paramount in making intelligent business decisions about the proper amount of components to obtain commitments for in any given period.

Problem Solved by the Invention:

1. How to compute the expected component surplus at the end of a particular planning period
2. How to compute the value of expected erosion costs for a particular set of components in a planning period

3. How to compute the total expected erosion costs in a company for a particular planning period

The essence of this invention is the ability to solve the above problems in cases where there is a plurality of products and components.

Indication of Novelty

1) This invention represents a novel application of the invention disclosed in [1] which enables highly scalable, high performance computation of expected erosion, cancellation and expediting.
2) The method of computing expected erosion costs in terms of expected production computations is novel.

Detailed Description of a Preferred Embodiment

Computing expected surplus

The first part of the invention is the computation of surplus components (expected excess number of components remaining at the end of a period). This is illustrated in Figure 1. The method consists of the following steps:

In step 101, we obtain the product and component data necessary for the calculation. These data include all the data required for mean production computation as disclosed in [1]. In addition, it is necessary to specify the levels d_i^0 for all components i of interest. If d_i represents the maximum available number of units component i (i.e. the expeditable amounts specified for the mean production computations) then $d_i - d_i^0$ represent the number of component i that can be liquidated or returned at no cost. The value d_i^0 may be equal to d_i for some or all the components.

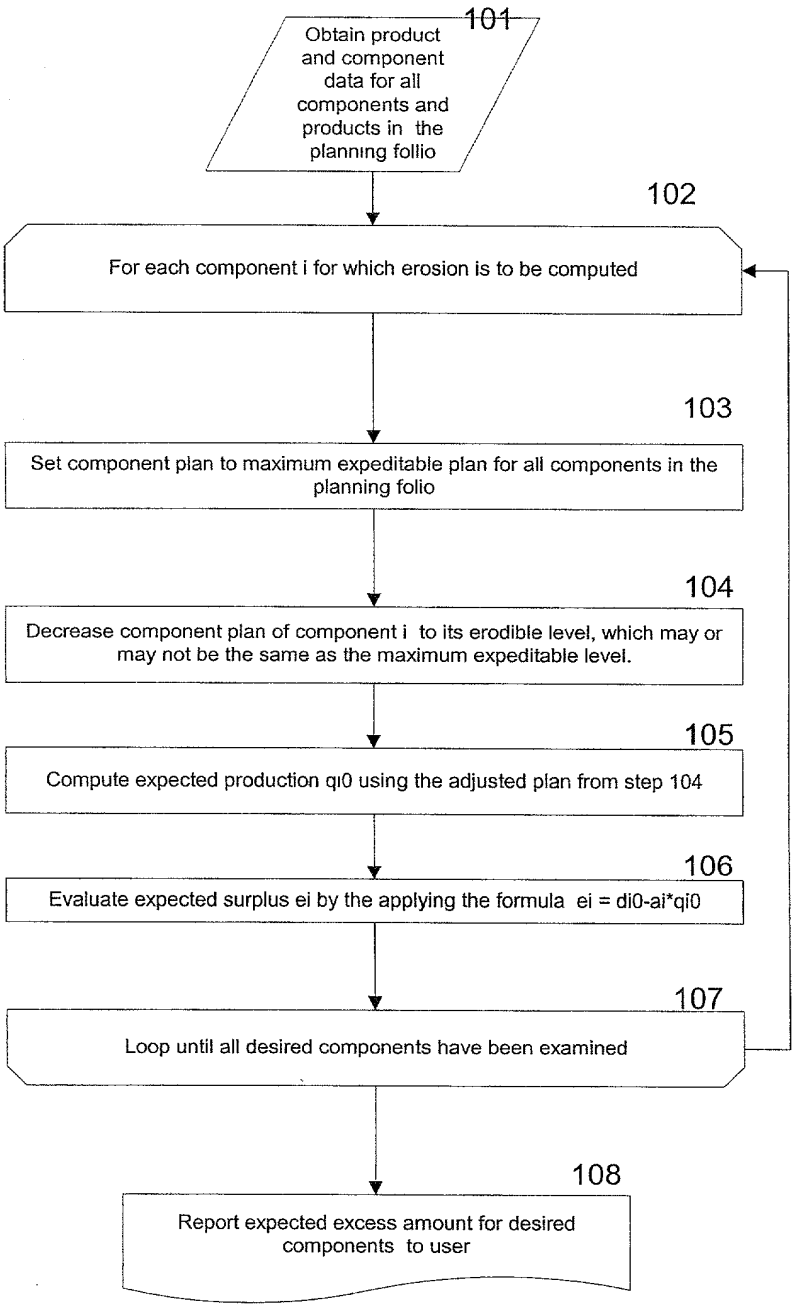

Figure 1 Evaluation of expected number of erodible components

In step 102-107 we loop over the subset of components for which the calculation is desired.

In step 103 we set the component plans d_i to their original values.

In step 104 we modify the component plan for component i (i.e., d_i) to d_i^0.

In step 105 we employ the method disclosed in [1] to compute the expected production for the above modified problem setup. Note that other methods, such as Monte Carlo simulation, which can compute the expected production in a planning period, could also be used. This invention is not limited to the method disclosed in [1].

In step 106 we use the formula $e_i = d_i^0 - a_i \cdot q_i^0$ to compute the expected number of erodible components i. In this formula d_i^0 is the erosion level, a_i is the vector of connect rates (as described in [1]) for components i, and q_i^0 is the mean production computed in step 105.

In step 108 the expected excess is reported to the user or returned to the system for further use.

Computing expected erosion costs per component

The second part of the invention is estimating the expected financial cost to the company of having leftover inventory for a subset of components. This consists of the steps described in Figure 2

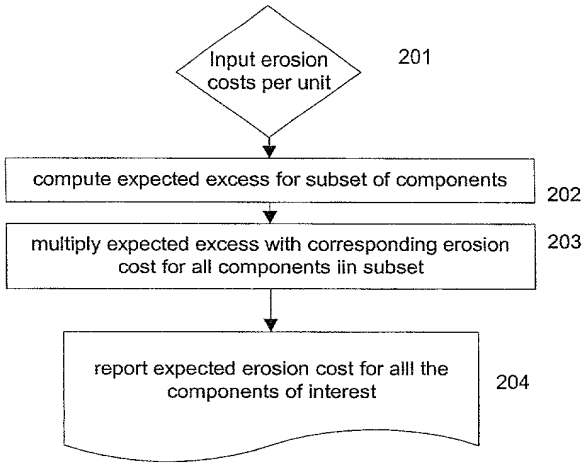

Figure 2 Computing expected erosion costs

In step 201 we input the erosion costs per unit surplus.

In step 202 we evaluate the expected surplus as described in process 100. This step may be done in parallel.

In step 203 we multiply the expected surplus by the per-unit costs to obtain the expected erosion cost for each component of interest.

In step 204 we report the expected erosion cost for each component of interest.

Computing total expected erosion cost

The third part of this invention concerns estimating the total erosion cost a company should expect given a certain plan for component availability. This process is illustrated in Figure 3.

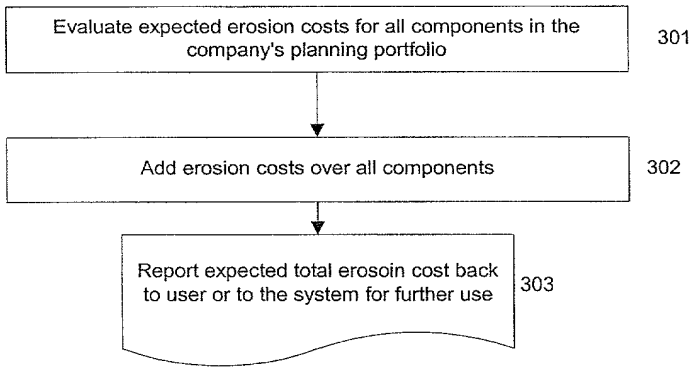

Figure 3 Evaluation of expected total erosion cost.

The steps are:

In step 301 the component-by-component expected erosion costs are evaluated for all components of the planning portfolio. This step may be done in parallel.

In step 302 the numbers obtained in step 301 are summed.

In step 303 the single expected erosion cost number is reported back to the user, or stored for further processing by the system.

Finally, the method of computing erosion costs can be used as the basis for computing expected cancellation and expediting fees following equations 4.4 and 5.2 in [2].

Technical field of the invention

This invention relates generally to the field of manufacturing resource planning. More specifically it relates to risk management in the procurement of unfinished goods.

Statutory Classification of the invention:

This invention represents a system and method.

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References

1. Dagum, P., M. Goldbach, and B. Kralik, *Method and apparatus for the estimation of mean production for assemble-to-order manufacturing operations.* U.S. Provisional Patent Application No. 60/213,189, filed 6/21/2000.

2. Kralik, B. and M. Goldbach, *Erosion, Cancellation and Expediting under Distributed Production.* 2000: Rapt Inc.**TR-15-99**

RAP-105/PROV

Provisional Patent Application of

Balazs Kralik, Michael Goldbach, and Paul Dagum

for

Method and Business Process for Estimation of Erosion Costs in Assemble-to-Order Manufacturing Operations

CROSS-REFERENCE TO RELATED APPLICATIONS

This invention is related to Provisional Application No. 60/213,189, filed 6/21/2000, which is herein incorporated by reference. It is also related to U.S. Patent Application Nos. 09/412,560, filed 10/5/1999, and 09/491,461, filed 1/26/2000, both of which are herein incorporated by reference.

FIELD OF THE INVENTION

This invention relates generally to manufacturing resource allocation. More particularly, it relates to risk management in the procurement of unfinished goods by estimation of expected excess component inventory and erosion costs.

SUMMARY

An assemble-to-order (ATO) manufacturing process is one in which products are manufactured from raw components only as orders are received. While ATO processes are more efficient than manufacture-to-stock (MTS) operations, in which products are produced before demand is known, they are not without significant risks. If component availability is below what is required to meet product demand, then fulfillment of orders is either delayed or fails. Conversely, if excess component inventory remains after demand is met, the excess is either returned to the supplier, usually at a loss, or held until future need. Thus any component inventory that does not

exactly meet demand incurs some financial loss for the manufacturer. Being able to predict the excess component inventory and associated financial loss allows a manufacturer to make more informed business decisions.

The present invention addresses this need by providing a novel method for estimating component surplus and expected erosion costs. The erosion cost is simply the cost incurred for component inventory that exceeds the inventory required to meet demand. Estimation of erosion cost is particularly complicated when a large number of products are made from a large number of the same components, and thus the present invention is particularly advantageous under such conditions.

Full description of the method of the present invention is contained within the following attached appendices as well as in the patent applications incorporated by reference above:

Appendix A: Invention Disclosure ID-06-00, "Method and business process for the estimation of erosion costs in assemble-to-order manufacturing operations," Balazs Kralik et al.

Appendix B: Technical Report TR-15-00, "Erosion, Cancellation and Expediting under Distributed Production," Balazs Kralik and Michael Goldbach.

In addition to a business process and computer-implemented method, the present invention provides a system for computing expected component surplus and erosion costs. The system is preferably a computer system containing a memory, input/output devices, and processing means for implementing the method.

The present invention also provides a computer-readable medium of instructions executable by the computer to perform method steps for the method described above. The present invention applies equally regardless of the particular type of signal bearing media used. Examples of computer-readable media include: recordable-type media such as floppy disks and CD-ROMs and transmission-type media such as digital and analog communication links.

It will be clear to one skilled in the art that the above embodiment may be altered in many ways without departing from the scope of the invention.